Loop Impact in the Three-loop Limits-to-Growth Model

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Consider a one stock limits-to-growth model with an added draining process, preventing the stock from achieving the natural carrying capacity, figurefigure1.fig. The model has one reinforcing loop driving the growth and two balancing loops in opposition. Loop impacts are computed numerically using Hayward & Boswell's (2014) method and confirmed analytically using pathway differentiation (Hayward & Roach, 2017; 2019).

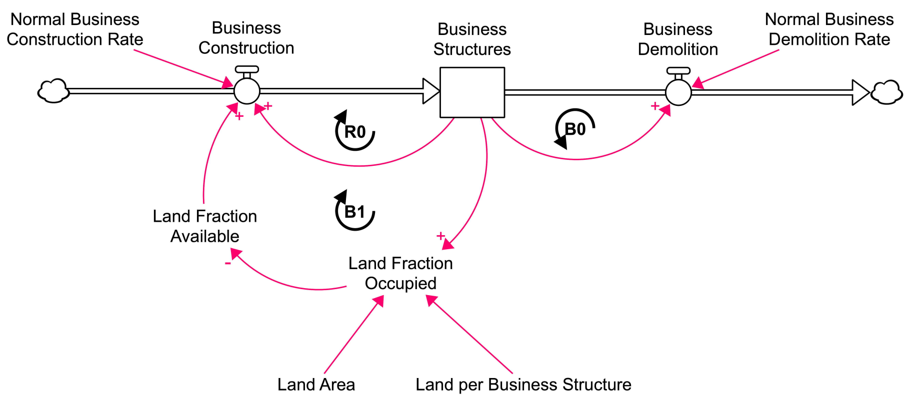

Fig. 1: Three-loop Limits-to-Growth model.

Let x represent business structures, then the underlying differential equation for the stock-flow system is written as:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = F = f(x) - g(x) = ax\left(1 - \frac{x}{m}\right) - bx \tag{1}$$

where a is the business construction rate, b is the business demolition rate, and m is the land area divided by land per business structure. f is business construction, g is business demolition and F is the net flow. To compute the loop impacts, rewrite the differential equation (1) in causally connected notation where the three pathways are indicated by underlined subscripts:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = ax_{\underline{f}}\left(1 - \frac{x_{\underline{OAf}}}{m}\right) - bx_{\underline{g}} \tag{2}$$

where O is the land fraction occupied and A the land fraction available.

Using the pathway differentiation method of Hayward & Roach (2017) pp. 16–17, the loop impacts become:

$$\mathrm{I}(R_0) = \mathrm{I}_{\underline{f}} = \frac{\partial F}{\partial x}\Bigg\|_{\underline{f}} = a\left(1 - \frac{x}{m}\right) \tag{3}$$

$$\mathrm{I}(B_1) = \mathrm{I}_{\underline{OAf}} = \frac{\partial F}{\partial x}\Bigg\|_{\underline{OAf}} = -\frac{ax}{m} \tag{4}$$

$$\mathrm{I}(B_0) = \mathrm{I}_{\underline{g}} = \frac{\partial F}{\partial x}\Bigg\|_{\underline{g}} = -b \tag{5}$$

where the partial derivatives are on the pathway subscripted version of x. The loop impacts are plotted in figure 2 with the loop transitions marked. The same loop transitions are marked on the stock graph, figure 3.

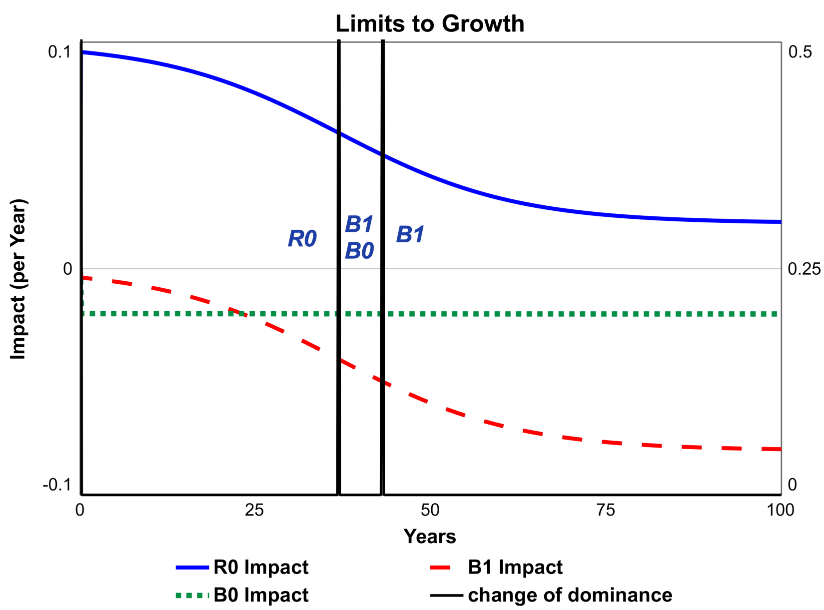

Fig. 2: Three-loop Limits-to-Growth loop impacts.

The first transition point is where the sum of the two balancing loops begins to exceed the reinforcing loop $\mathrm{I}(B_0) + \mathrm{I}(B_0) \geq \mathrm{I}(R_0)$. This transition occurs at the inflexion point $x/m \geq \frac{1}{2}(1 - b/a)$ (from equations 3–5). To the left of the point the stock is accelerating, to the right it is decelerating.

The second transition point is when loop B_1 can dominate in its own right. $\mathrm{I}(B_0) \geq \mathrm{I}(R_0)$, that is $x/n \geq \frac{1}{2}$. When b, the gain of B_0, is zero, the two transition points merge.

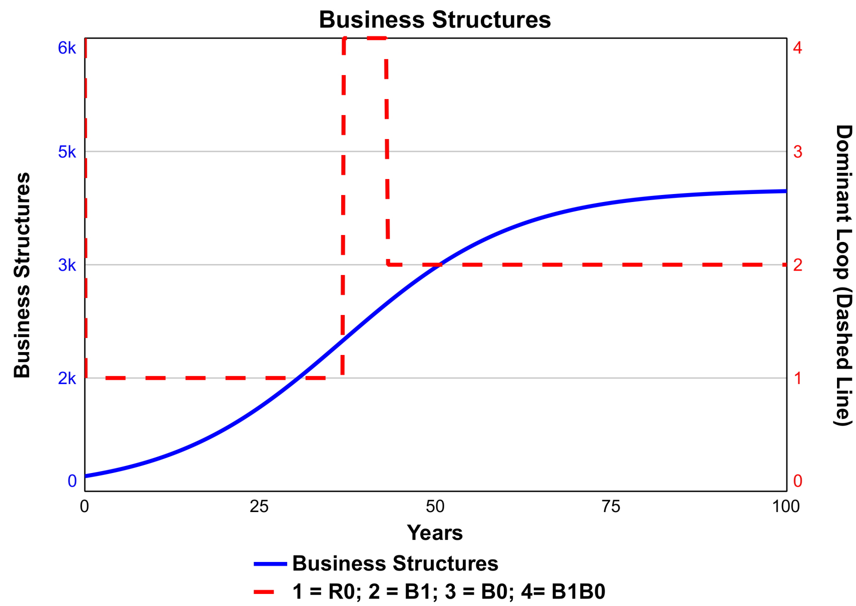

Fig. 3: Three-loop Limits-to-Growth loop dominance.

References

Hayward J, Boswell GP. 2014. Model behaviour and the concept of loop impact: A practical method. *System Dynamics Review*, **30(1)**, 29–57.

Hayward J, Roach PA. 2017. Newton's laws as an interpretive framework in system dynamics. *System Dynamics Review*, **33(3–4)**, 183–218.

Hayward J, Roach PA., 2019. The concept of force in population dynamics. *Physica A: Statistical Mechanics and its Applications*, **531**, 121736.